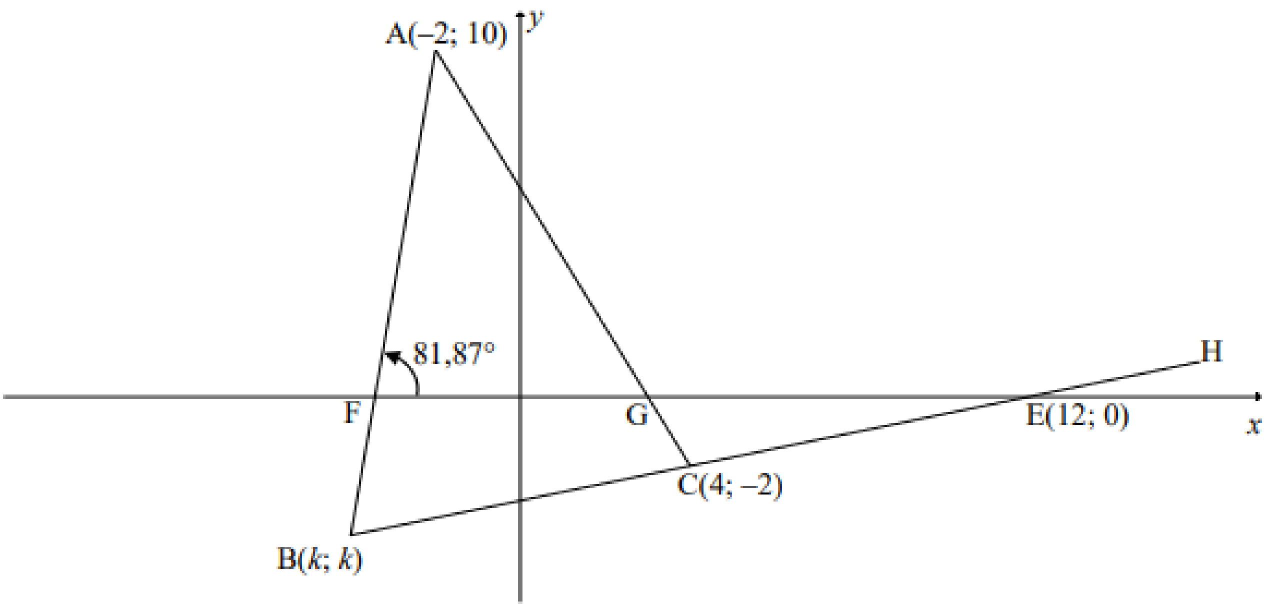
ANALYTICAL GEOMETRY MEMO 2022 (NOVEMBER 2021)

QUESTION/*VRAAG* 3

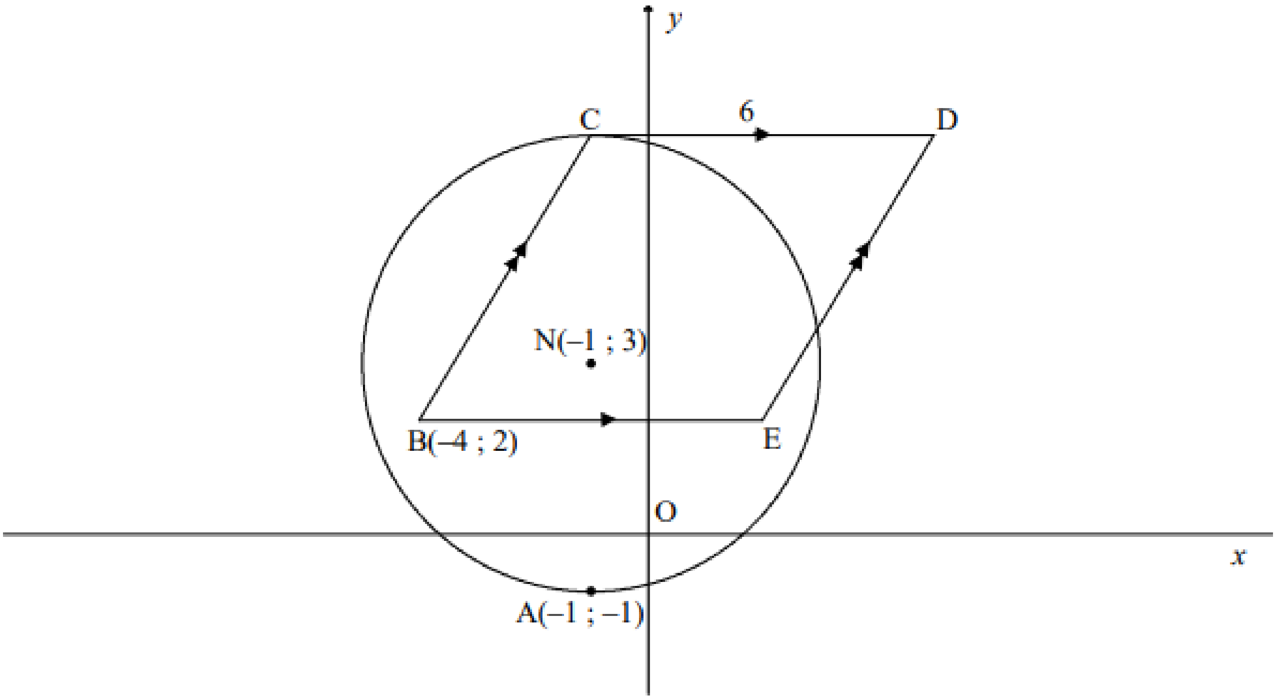


3.1.1	$m_{BE} = m_{CE} = \frac{0-(-2)}{12-4}$ **OR/*OF*** $m_{BE} = m_{CE} = \frac{-2-0}{4-12}$ $= \frac{1}{4}$ $= \frac{1}{4}$	✓ substitution C & E ✓ answer (2)
3.1.2	$m_{AB} = \tan 81{,}87°$ $m_{AB} = 7$ Answer only: Full marks *Slegs antw: Volpunte*	✓ substitution ✓ answer (2)
3.2	$y = mx + c$ or $y - y_1 = m(x - x_1)$ $0 = \frac{1}{4}(12) + c$ or $y - 0 = \frac{1}{4}(x - 12)$ $c = -3$ $y = \frac{1}{4}x - 3$ $y = \frac{1}{4}x - 3$ **OR/*OF*** $y = mx + c$ or $y - y_1 = m(x - x_1)$ $-2 = \frac{1}{4}(4) + c$ or $y-(-2) = \frac{1}{4}(x - 4)$ $c = -3$ $y = \frac{1}{4}x - 3$ $y = \frac{1}{4}x - 3$	✓ substitution of E ✓ answer (2) ✓ substitution of C ✓ answer (2)

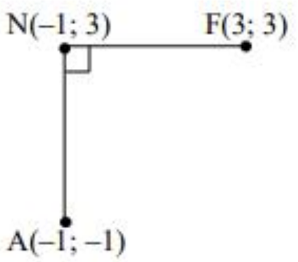
3.3.1	$y=\frac{1}{4}x-3$ $k=\frac{1}{4}k-3$ $\frac{3}{4}k=-3$ $k=-4$ $\therefore B(-4;-4)$	✓ substitution ✓ answer (2)
	OR/*OF* $m_{BE}=\frac{1}{4}$ $\frac{0-k}{12-k}=\frac{1}{4}$ $-4k=12-k$ $k=-4$ $\therefore B(-4;-4)$ **OR/*OF*** $m_{BE}=\frac{1}{4}$ $\frac{k}{k-12}=\frac{1}{4}$ $4k=k-12$ $k=-4$	✓ substitution ✓ answer (2)
	OR/*OF* $m_{AB}=\tan 81{,}87°$ $m_{AB}=7$ $m_{AB}=\frac{10-k}{-2-k}$ $7(-2-k)=10-k$ $-14-7k=10-k$ $-6k=24$ $k=-4$ $\therefore B(-4;-4)$	✓ substitution ✓ answer (2)
	OR/*OF* EB: $y=\frac{1}{4}x-3$ and AB: $y=7x+24$ $\frac{1}{4}x-3=7x+24$ $\frac{27}{4}x=-27$ $x=k=-4$ $\therefore B(-4;-4)$	✓ equating EB & AB ✓ answer (2)

3.3.2	In ΔAFG: $m_{AC} = \frac{10-(-2)}{-2-4} = -2$ $\tan\theta = m_{AC} = -2$ $\theta = 180° - 63,43...°$ $\therefore \theta = 116,57°$ $\therefore \hat{A} = 116,57° - 81,87°$ [ext ∠ of Δ] $\therefore \hat{A} = 34,70°$ **OR/*OF*** In ΔABC: $a = BC = 2\sqrt{17}; b = AC = 6\sqrt{5}; c = AB = 10\sqrt{2}$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $\left(2\sqrt{17}\right)^2 = \left(6\sqrt{5}\right)^2 + \left(10\sqrt{2}\right)^2 - 2(6\sqrt{5})(10\sqrt{2}).\cos A$ $\cos A = \frac{\left(6\sqrt{5}\right)^2 + \left(10\sqrt{2}\right)^2 - \left(2\sqrt{17}\right)^2}{2\left(6\sqrt{5}\right)\left(10\sqrt{2}\right)}$ $= 0,822...$ $\therefore A = 34,7°$	✓ $m_{AC} = -2$ ✓ $\tan\theta = -2$ ✓ $\theta = 116,57°$ ✓ answer (4) ✓ all 3 lengths ✓ substitution into the correct cosine rule ✓ cos A subject ✓ answer (4)
3.3.3	$M\left(\frac{12+(-2)}{2}; \frac{10+(0)}{2}\right)$ Diagonals intersect at the point (5 ; 5)	✓ x-value ✓ y-value (2)
3.4.1	BE = ET $4\sqrt{17} = \sqrt{(12-p)^2 + (0-p)^2}$ $\left(4\sqrt{17}\right)^2 = \left(\sqrt{(12-p)^2 + (0-p)^2}\right)^2$ $272 = 144 - 24p + p^2 + p^2$ $p^2 - 12p - 64 = 0$ $(p-16)(p+4) = 0$ $\therefore p = 16$ or $p = -4$ (n.a.) $\therefore$T(16; 16)	✓ substitution of E & T ✓ equating ✓ standard form ✓ factors ✓ $p = 16$ (5)
3.4.2a	$(x-12)^2 + y^2 = \left(4\sqrt{17}\right)^2 = 272$	✓ LHS ✓ RHS (2)
3.4.2b	$m_{radius} = \frac{1}{4}$ $m_{tangent} = -4$ $y = -4x + c$ $-4 = -4(-4) + c$ $c = -20$ $y = -4x - 20$ **OR/*OF*** $y - y_1 = -4(x - x_1)$ $y - (-4) = -4(x - (-4))$ $y = -4x - 20$	✓ $m_{tangent}$ ✓ substitution of B ✓ equation (3)
		[24]

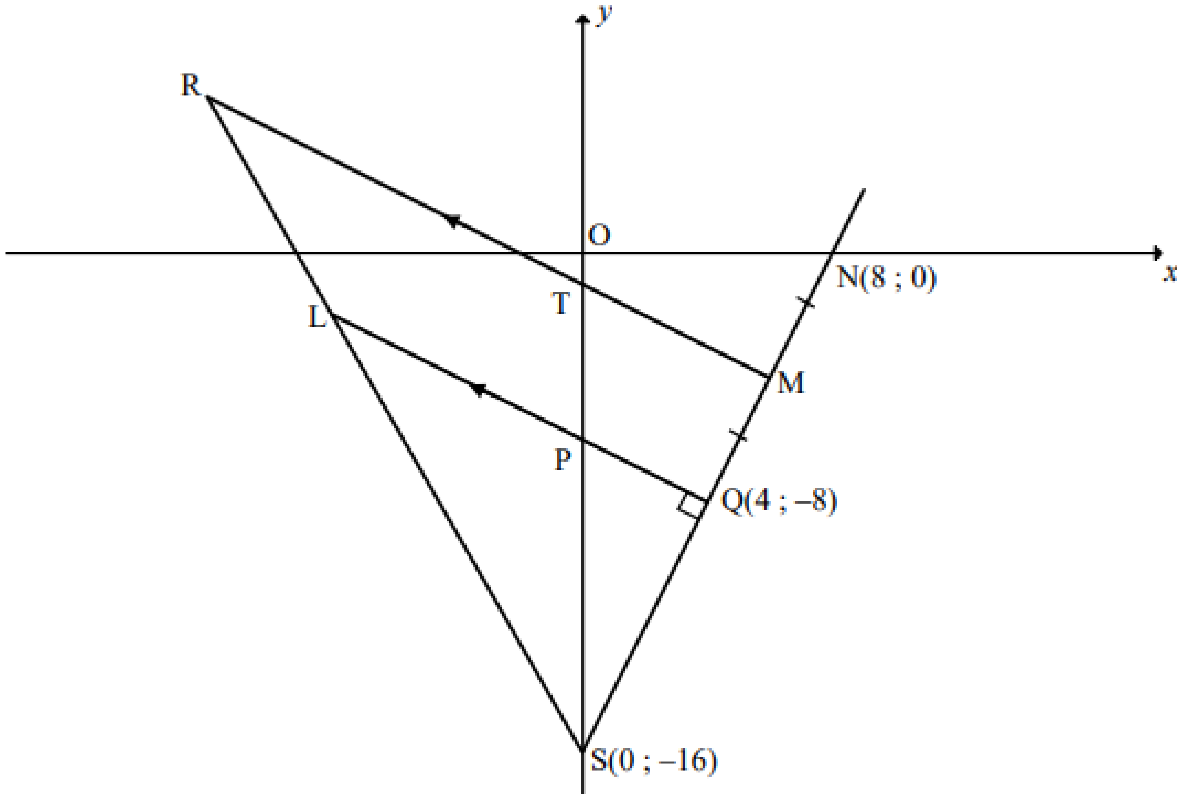
QUESTION/*VRAAG* 4



4.1	Radius = 4 units/*eenhede*	✓ answer (1)
4.2.1	$CD \perp CN$ $\therefore C(-1\,;\,7)$	✓x value ✓ y value (2)
4.2.2	CD = 6 units $\therefore D(5\,;\,7)$	✓x value ✓ y value (2)
4.2.3	$\perp h = 5$ units DC = 6 units Area $\Delta BCD = \frac{1}{2}(6)(5)$ $= 15\text{ units}^2$ **OR/*OF*** $\perp h = 5$ units DC = 6 units Area $\Delta BCD = \frac{1}{2}[\text{Area of } \|^m]$ $= \frac{1}{2}[(5)(6)]$ $= 15\text{ units}^2$	✓ $\perp h = 5$ units ✓ substitution into Area formula ✓ answer (3) ✓ $\perp h = 5$ units ✓ substitution into Area formula ✓ answer (3)

	OR/*OF* Let angle of inclination of BC = α $\tan\alpha = \frac{5}{3}$ $\alpha = 59{,}036...^\circ$ $B\hat{C}D = 180^\circ - \alpha$ $B\hat{C}D = 180^\circ - 59{,}036...^\circ$ $B\hat{C}D = 120{,}96^\circ$ Area $\Delta BCD = \frac{1}{2}\left(\sqrt{34}\right)(6)\sin 120{,}96^\circ$ $= 15 \text{ units}^2$	✓ $B\hat{C}D = 120{,}96^\circ$ ✓ substitution into Area rule ✓ answer (3)
4.3.1	$M(3\ ;-1)$ [reflection of $N(-1\ ;3)$ about the line $y = x$] $\therefore MN = \sqrt{(3-(-1))^2 + (-1-3)^2}$ $MN = \sqrt{32} = 4\sqrt{2} = 5{,}66$ units	✓ coordinates of M (A) ✓ substitution of M&N ✓ answer (3)
4.3.2	$M(3\ ;-1)$ $m_{MN} = \frac{3-(-1)}{-1-3} = -1$ MN: $-1 = -(3) + c$ or $y - 3 = -1(x+1)$ $c = 2$ $y - 3 = -x - 1$ $\therefore y = -x + 2$ $y = -x + 2$ $x = -x + 2$ $2x = 2$ $x = 1$ $\therefore y = 1$ midpoint $(1\ ;1)$	✓ equation of MN ✓ equating AF & MN ✓x value ✓ y value (4)
	OR/*OF*  $N(-1\ ;3)$ $y_F = y_N = 3$ Reflected about $y = x$ $\therefore F(3\ ;3)$ midpoint $\left(\frac{-1+3}{2};\frac{-1+3}{2}\right) = (1\ ;1)$	 ✓✓ coordinates of F ✓x value ✓ y value (4)

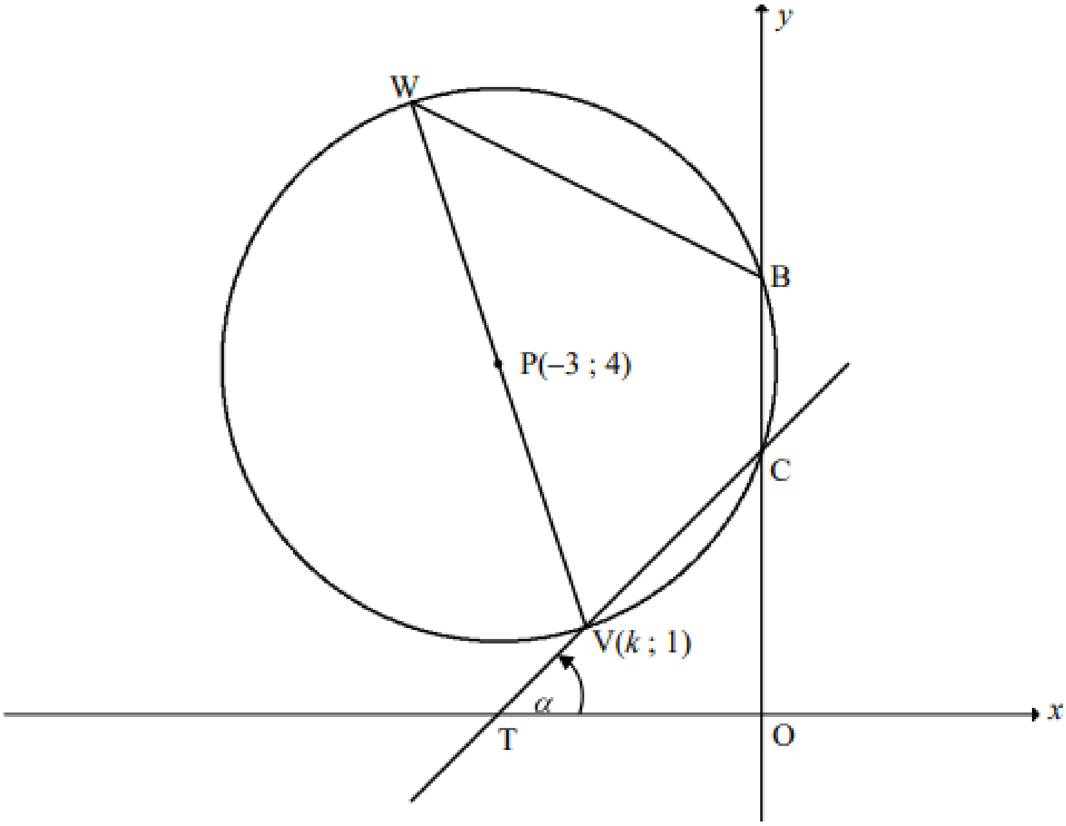
QUESTION/VRAAG 3



3.1	$M\left(\frac{4+8}{2};\frac{-8+0}{2}\right)$ $M(6;-4)$	✓ x_M ✓ y_M (2)
3.2	$m_{NS}=\frac{0-(-16)}{8-0}$ or $m_{NQ}=\frac{0-(-8)}{8-4}$ or $m_{QS}=\frac{-8-(-16)}{4-0}$ $=2$	✓ subst N and Q or N and Q or Q and S into gradient formula ✓ answer (2)
3.3	$m_{LQ}\times 2=-1$ [LQ ⊥ NS] $\therefore m_{LQ}=-\frac{1}{2}$ $-8=-\frac{1}{2}(4)+c$ **OR** $y+8=-\frac{1}{2}(x-4)$ $c=-6$ $y+8=-\frac{1}{2}x+2$ $\therefore y=-\frac{1}{2}x-6$	✓ m_{LQ} ✓ substitution of Q ✓ calculation of c or simplification (3)
3.4	OS is the radius of circle passing through S $(x-0)^2+(y-0)^2=(16)^2$ $x^2+y^2=256$ Answer only: Full marks	✓ identifying radius = 16 ✓ Equation of circle (2)

3.5	$m_{RM} = m_{LQ} = -\frac{1}{2}$ [RM ∥ LQ] $-4 = -\frac{1}{2}(6) + c$ **OR** $y + 4 = -\frac{1}{2}(x-6)$ $c = -1$ $y + 4 = -\frac{1}{2}x + 3$ $\therefore y = -\frac{1}{2}x - 1$ $T(0;-1)$	✓ m_{RM} ✓ substitution of $M(6;-4)$ ✓ coordinates of T (3)
3.6	$T(0;-1)$, $P(0;-6)$ and $S(0;-16)$ $\therefore$ PS = 10 units and TS = 15 units $\frac{LS}{RS} = \frac{PS}{TS} = \frac{2}{3}$ [prop theorem; RM ∥ LP] **OR** [line ∥ one side of Δ/*lyn* ∥ *een sy v Δ*] Answer only: Full marks **OR** M(6 ; –4), Q(4 ; –8) and S(0 ; –16) $MS = \sqrt{180} = 6\sqrt{5}$ and $QS = \sqrt{80} = 4\sqrt{5}$ $\frac{LS}{RS} = \frac{QS}{MS} = \frac{2}{3}$ [prop theorem; RM ∥ LQ] **OR** [line ∥ one side of Δ/*lyn* ∥ *een sy v Δ*] Answer only: Full marks	✓ PS = 10 units ✓ TS = 15 units ✓ answer (3) ✓ $MS = 6\sqrt{5}$ units ✓ $QS = 4\sqrt{5}$ units ✓ answer (3)
3.7	area of PTMQ = area of ΔTSM – area of ΔPSQ $= \frac{1}{2}ST. \perp h_M - \frac{1}{2}.PS. \perp h_Q$ $= \frac{1}{2}(15)(6) - \frac{1}{2}(10)(4)$ $= 45 - 20$ = 25 square units **OR** $TM = \sqrt{45} = 3\sqrt{5} = 6{,}71$ $MQ = \sqrt{20} = 2\sqrt{5} = 4{,}47$ $PQ = \sqrt{20} = 2\sqrt{5} = 4{,}47$ area of trapezium PTMQ = $\frac{1}{2}(3\sqrt{5} + 2\sqrt{5})(2\sqrt{5})$ $= \frac{1}{2}(5\sqrt{5})(2\sqrt{5})$ = 25 square units	✓ area of ΔTSM – area of ΔPSQ ✓ area ΔTSM = 45 ✓ area ΔPSQ = 20 ✓ answer (4) ✓ $TM = 3\sqrt{5}$ $MQ = 2\sqrt{5}$ $PQ = 2\sqrt{5}$ ✓ area of trapezium = $\frac{1}{2}$ (sum of ∥sides)(height) ✓ substitute into formula ✓ answer (4)

QUESTION 4

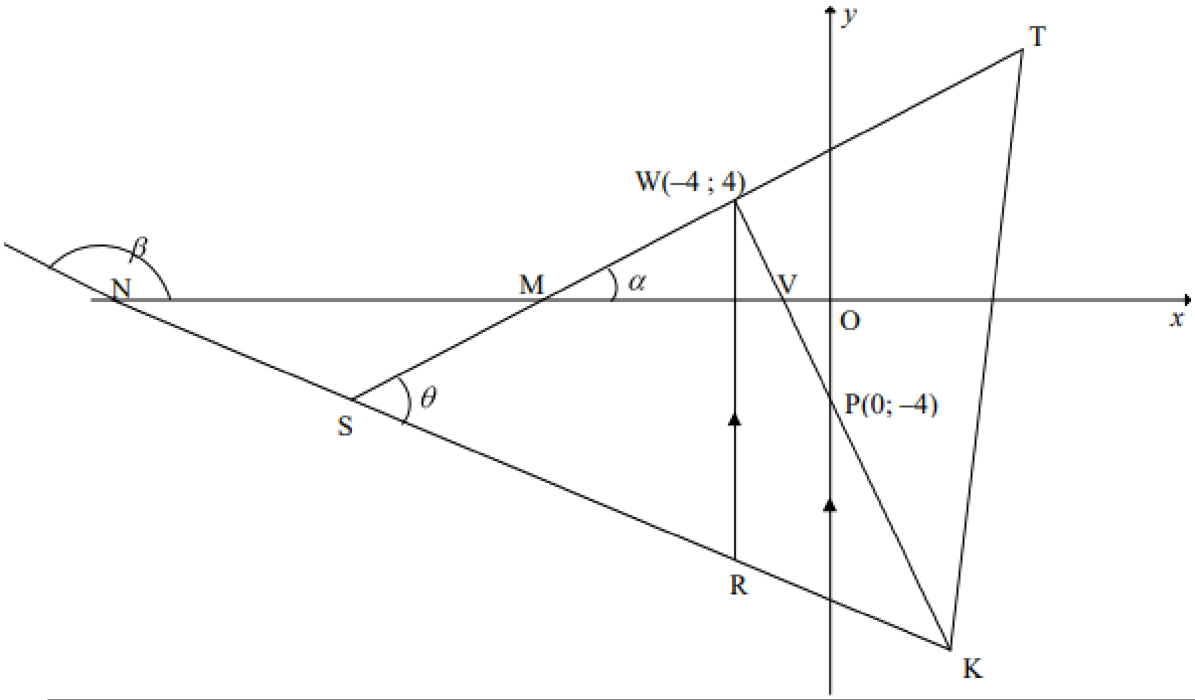


4.1	$PV = r = \sqrt{10}$ $PV = \sqrt{(k-(-3))^2 + (1-4)^2} = \sqrt{10}$ $(PV)^2 = (k-(-3))^2 + (1-4)^2 = 10$ $k^2 + 6k + 9 + 9 = 10$ **OR** $(k+3)^2 + 9 = 10$ $k^2 + 6k + 8 = 0$ $(k+3)^2 = 1$ $(k+4)(k+2) = 0$ $k+3 = 1$ or $k+3 = -1$ $k = -4$ or $k = -2$ $\therefore k = -2$	✓ $PV = r = \sqrt{10}$ ✓ substitution into distance formula ✓ standard form ✓ factors ✓ answer (5)
4.2	$x^2 + 6x + y^2 - 8y + 15 = 0$ y-intercepts: $(0)^2 + 6(0) + y^2 - 8y + 15 = 0$ $(y-3)(y-5) = 0$ $y_C = 3$ or $y_B = 5$ $\therefore$ BC $= 2$ units	✓ $x = 0$ ✓ factors ✓ both values ✓ answer (4)

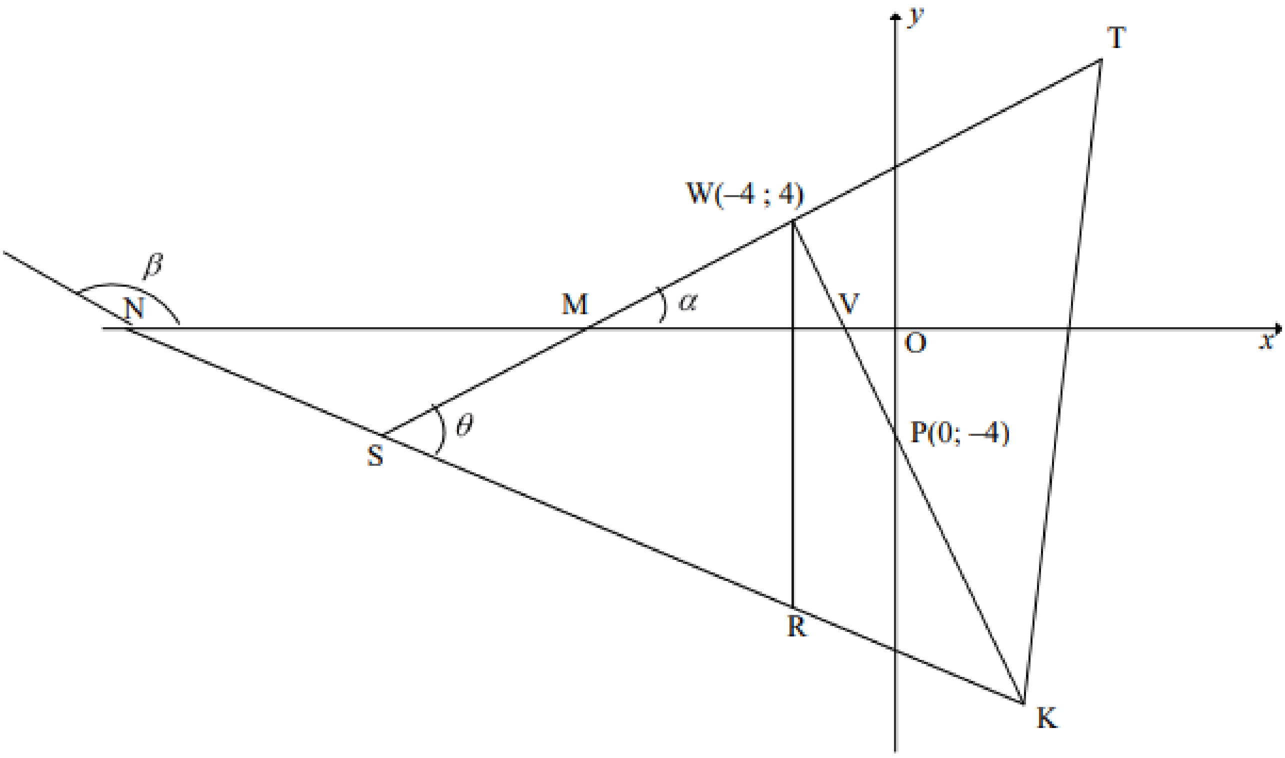
4.3.1	$m_{TC} = \frac{3-1}{0-(-2)}$ $= 1$ $\tan\alpha = 1$ $\therefore \alpha = 45°$ **OR** $y = mx + 3$ $1 = m(-2) + 3$ $m_{TC} = 1$ $\tan\alpha = 1$ $\therefore \alpha = 45°$	✓ substitution into gradient formula ✓ $\tan\alpha = 1$ ✓ answer (3) ✓ substitution into equation of a line ✓ $\tan\alpha = 1$ ✓ answer (3)
4.3.2	$B\hat{C}V = 135°$ [ext ∠ of Δ/*buite* ∠ v Δ] $\therefore V\hat{W}B = 45°$ [opp ∠s of cyclic quad/*teenoorst. ∠e v kvh*] Answer only: Full marks **OR** $T\hat{C}O = 45°$ [∠s of Δ/∠*e* v Δ] $\therefore V\hat{W}B = 45°$ [ext ∠s of cyclic quad/*buite* ∠ *v kvh*] Answer only: Full marks	✓ $B\hat{C}V = 135°$ ✓ answer (2) ✓ $T\hat{C}O = 45°$ ✓ answer (2)
4.4.1	$Q(-3;-2)$	✓ x_Q ✓ y_Q (2)
4.4.2	$(x+3)^2 + (y+2)^2 = 10$	✓ LHS ✓ RHS (2)
4.4.3	$x = -2$ or $x = -4$	✓ $x = -2$ ✓ $x = -4$ (2)
		[20]

NOVEMBER 2020

QUESTION/*VRAAG* 3



3.1	$m_{WP} = \frac{4-(-4)}{-4-0} = \frac{8}{-4}$ $m_{WP} = -2$	✓ substitution of W and P ✓ m_{WP} (2)
3.2	$m_{ST} = \frac{1}{2}$ (given) $(m_{WP})(m_{ST}) = (-2)\left(\frac{1}{2}\right)$ $= -1$ $\therefore ST \perp WP$	✓ $(m_{WP})(m_{ST})$ ✓ $(m_{WP})(m_{ST}) = -1$ (2)
3.3	$5y + 2x + 60 = 0$ $\therefore y = -\frac{2}{5}x - 12$ $-\frac{2}{5}x - 12 = \frac{1}{2}x + 6$ $-4x - 120 = 5x + 60$ $9x = -180$ $x = -20$ $\therefore y = -\frac{2}{5}(-20) - 12$ $\therefore y = -4$ $\therefore S(-20; -4)$ **OR**	✓ equating ✓ x value ✓ substitution ✓ y value (4)



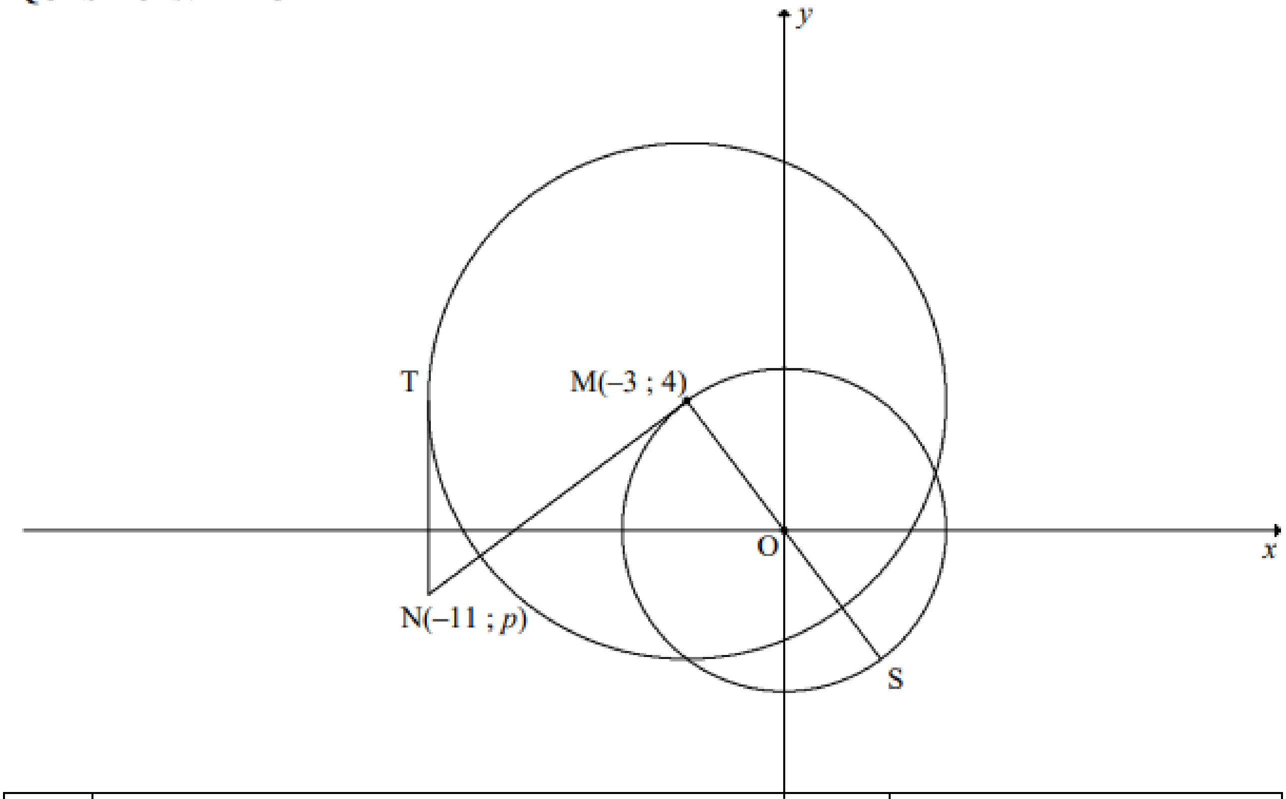
	$5y+2x+60=0$ $5\left(\frac{1}{2}x+6\right)+2x+60=0$ $\frac{5}{2}x+30+2x+60=0$ $\frac{9}{2}x=-90 \qquad \therefore x=-20$ $\therefore y=-\frac{2}{5}(-20)-12$ $\therefore y=-4$ $\therefore$ S(–20; –4) **OR** $5y+2x=-60$ ....(1) $2y-x=12$ ....(2) $(1)+2(2): 9y=-36$ $y=-4$ $2(-4) - x=12$ $x=-20$	✓ substitution ✓ x value ✓ substitution ✓ y value (4) ✓ adding ✓ y value ✓ substitution ✓ x value (4)

3.4	$y=-\frac{2}{5}(-4)-12$ OR $5y+2(-4)+60=0$ $y=-\frac{52}{5}$ $\therefore R\left(-4;-\frac{52}{5}\right)$ OR R(–4 ; –10,4) $\therefore WR=4-\left(-\frac{52}{5}\right)$ OR $WR=\sqrt{(-4-(-4))^2+(4-\left(-\frac{52}{5}\right))^2}$ $\therefore WR=\frac{72}{5}$ units or $WR=14\frac{2}{5}$ units **OR** WR = ST – SK $=\frac{1}{2}x+6-\left(-\frac{2}{5}x-12\right)$ $=\frac{9}{10}x+18$ $=\frac{9}{10}(-4)+18$ $=14,4$ units	✓ substitution ✓ y value ✓ method or subst into distance formula ✓ answer (4) ✓ substitution ✓ simplification ✓ subst $x=-4$ ✓ answer (4)
3.5	$m_{SK}=-\frac{2}{5}$ $\beta=158,19...°$ (Ref. ∠ =21, 801…°) $M\hat{N}S=21,80...°$ $m_{ST}=\frac{1}{2}$ $N\hat{M}S=26,56...°$ $\theta=21,80...°+26,56...°$ [ext ∠ of Δ] $\theta=48,366...°=48,37°$	✓ m_{SK} ✓ size of β ✓ size of $N\hat{M}S$ ✓ method ✓ answer (5)
3.6	In ΔSRW: $\perp h=-4-(-20)$ $\perp h=16$units Area $\Delta SRW=\frac{1}{2}(\perp h)(WR)$ $=\frac{1}{2}(16)\left(\frac{72}{5}\right)$ $=115,2$ square units Area SWRL = 2Area ΔSRW =2(115,2) = 230,4square units **OR**	✓ $\perp h$ ✓ substitution ✓ area Δ ✓ answer (4)

3.4	$y = -\frac{2}{5}(-4) - 12$ OR $5y + 2(-4) + 60 = 0$ $y = -\frac{52}{5}$ $\therefore R\left(-4; -\frac{52}{5}\right)$ OR R(–4 ; –10,4) $\therefore WR = 4 - \left(-\frac{52}{5}\right)$ OR $WR = \sqrt{(-4-(-4))^2 + (4 - \left(-\frac{52}{5}\right))^2}$ $\therefore WR = \frac{72}{5}$ units or $WR = 14\frac{2}{5}$ units **OR** $WR = ST - SK$ $= \frac{1}{2}x + 6 - \left(-\frac{2}{5}x - 12\right)$ $= \frac{9}{10}x + 18$ $= \frac{9}{10}(-4) + 18$ $= 14{,}4$ units	✓ substitution ✓ y value ✓ method or subst into distance formula ✓ answer (4) ✓ substitution ✓ simplification ✓ subst $x = -4$ ✓ answer (4)
3.5	$m_{SK} = -\frac{2}{5}$ $\beta = 158{,}19...^\circ$ (Ref. $\angle = 21{,}801...^\circ$) $M\hat{N}S = 21{,}80...^\circ$ $m_{ST} = \frac{1}{2}$ $N\hat{M}S = 26{,}56...^\circ$ $\theta = 21{,}80...^\circ + 26{,}56...^\circ$ [ext $\angle$ of Δ] $\theta = 48{,}366...^\circ = 48{,}37^\circ$	✓ m_{SK} ✓ size of β ✓ size of $N\hat{M}S$ ✓ method ✓ answer (5)
3.6	In ΔSRW: $\perp h = -4 - (-20)$ $\perp h = 16$units Area $\Delta SRW = \frac{1}{2}(\perp h)(WR)$ $= \frac{1}{2}(16)\left(\frac{72}{5}\right)$ $= 115{,}2$ square units Area SWRL = 2Area ΔSRW $= 2(115{,}2)$ $= 230{,}4$ square units **OR**	✓ $\perp h$ ✓ substitution ✓ area Δ ✓ answer (4)

	In ΔSRW: $\perp h = -4-(-20)$ $\perp h = 16$units Area SWRL $= 16 \times \dfrac{72}{5}$ $= 230,40$ square units	✓ $\perp h$ ✓ ✓ substitution ✓ answer (4)
	OR $SW = \sqrt{(-20+4)^2 + (-4-4)^2} = 8\sqrt{5} = 17,89$ $SR = \sqrt{(-20+4)^2 + \left(-4+10\frac{2}{5}\right)^2} = \dfrac{16\sqrt{29}}{5} = 17,23$ Area SWRL $= 2\times$ Area ΔSRW $= 2\left(\frac{1}{2} SW \times SR \sin\theta\right)$ $= 2\left(\frac{1}{2} 8\sqrt{5} \times \frac{16\sqrt{29}}{5} \sin 48,37°\right)$ $= 230,41$square units	✓$SW = 8\sqrt{5}$ ✓$SR = \dfrac{16\sqrt{29}}{5}$ ✓substitution ✓answer (4)
		[21]

QUESTION/*VRAAG* 4



4.1	$x^2 + y^2 = r^2$ $\therefore r^2 = (-3)^2 + (4)^2 = 25$ $x^2 + y^2 = 25$	✓ substitution ✓ answer (2)
4.2	TM ⊥ TN [tangent ⊥ radius] T(–11 ; 4) $r = -3 - (-11) = 8$ $(x+3)^2 + (y-4)^2 = 64$	✓ $x_T = -11$ ✓ LHS ✓ RHS (3)
4.3	O (0 ; 0) and M(–3; 4) $m_{OM} = \frac{4-0}{-3-0} = -\frac{4}{3}$ OR $\frac{0-4}{0-(-3)} = -\frac{4}{3}$ $m_{NM} = \frac{3}{4}$ $y - 4 = \frac{3}{4}(x-(-3))$ **OR** $y = \frac{3}{4}x + c$ $y - 4 = \frac{3}{4}x + \frac{9}{4}$ $4 = \frac{3}{4}(-3) + c$ $\therefore y = \frac{3}{4}x + \frac{25}{4}$ $c = \frac{25}{4}$ $y = \frac{3}{4}x + \frac{25}{4}$	✓ $m_{OM} = -\frac{4}{3}$ ✓ $m_{NM} = \frac{3}{4}$ ✓ substitution of m and M ✓ equation (4)

4.4	N(−11 ; p) $y=\frac{3}{4}x+\frac{25}{4}$ $p=\frac{3}{4}(-11)+\frac{25}{4}$ OR $\frac{4-p}{-3-(-11)}=\frac{3}{4}$ $p=-2$ $\therefore N(-11;-2)$ $\frac{-3+x_S}{2}=0$ and $\frac{4+y_S}{2}=0$ $\therefore S(3;-4)$ $SN=\sqrt{(-11-3)^2+(-2-(-4))^2}$ $=10\sqrt{2}$ units or 14,14 units	✓ subst $x=-11$ into eq or gradient ✓ $p=-2$ ✓ x_S ✓ y_S ✓ answer (CA) (5)
4.5	$B(-2;5)$ $BM=\sqrt{2}$ units Radius of circle centred at M = 8 units $k=8-\sqrt{2}$ or $k=8+\sqrt{2}$ = 6,59 units = 9,41 units = 6,6 units = 9,4 units	✓ $\sqrt{2}$ ✓✓ $k=6,6$ ✓✓ $k=9,4$ (5)
		[19]